

Strongly interacting and highly entangled photons in asymmetric quantum well with resonant tunneling

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We propose an asymmetric quantum well structure to realize strong interaction between two slow optical pulses. The linear optical properties and nonlinear optical responses associated with cross-Kerr nonlinearity are analyzed. Combining the resonant tunneling and the advantages of inverted-Y type scheme, giant cross-Kerr nonlinearity can be achieved with vanishing absorptions. Based on the unique feature, we demonstrate that highly entangled photons can be produced and photonic controlled phase gate can be constructed. In this construction, the scheme is symmetric for the probe and signal pulses. Consequently, the condition of group velocity matching can be fulfilled by adjusting the initial electron distribution.

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I. INTRODUCTION

Photons are ideal carriers of quantum information as they do not interact strongly with their environment and can be transmitted over long distances [1, 2]. Realizing efficient nonlinear interactions between single photons is considered a key step toward all-optical quantum computation and quantum information processing. While nonlinear effect whereby one light beam influences another requires large numbers of photons or else photon will be confined in a high- Q cavity. Hence the major obstacle of constructing scalable and efficient quantum computation with photonic qubits is the absence of giant cross-Kerr nonlinearity capable of entangling pairs of photons. A promising avenue has been opened by studies of enhanced nonlinear coupling via electromagnetically induced transparency (EIT) [3, 4]. In a four-level N -type scheme, it was proposed that the ultrahigh sensitivity of EIT dispersion to the two-photon Raman detuning in the vicinity of an absorption minimum can be used to enhance cross-Kerr nonlinearity between two weak optical fields [4, 5]. Large cross-Kerr nonlinearity emerges when two optical pulses, a probe and a signal, interact for a sufficiently long time. This happens when their group velocities are both reduced and comparable [6–8]. In order to eliminate the mismatch between the slow group velocity of the probe pulse subject to EIT and that of the nearly free propagating signal pulse, versatile novel symmetric configurations have been suggested theoretically and experimentally to realize the polarization phase gate, including

the tripod configuration [9–13], inverted-Y-type configuration [14, 15], M-type atomic schemes [7, 8, 16, 17], and so on [18]. More recently, large and rapidly responding cross-Kerr nonlinearity and highly entangled photons have been demonstrated in resonant atomic scheme based on active Raman gain configurations [19–21].

Semiconductor heterostructures provide a potential energy well with a size comparable to the de Broglie wavelength, trapping the carriers in discrete energy levels resulting in objects with atom-like optical properties. Different from atomic system, the interaction between semiconductor heterostructures and optical fields is strongly enhanced with merits such as the large electric dipole moments due to the small effective electron mass. Moreover, the intersubband energies and the electron function symmetries can be engineered as desired in accordance with the requirement. These advantages create the opportunities of building opto-electron devices that harness atom physics. Another important motivation of such study comes from the drastic increase in applications because of the wide-spread use of semiconductor components in optoelectronics and quantum information science. As a consequence, there has been a fast growth of research activity aimed at studying the quantum interference effects in semiconductors, for examples, the strong EIT [22], tunneling induced transparency (TIT) [23], ultrafast optical switching with Fano interference [24], slow light [25], etc [26–30]. Nonlinear optical properties in semiconductor heterostructures have also been paid much attention such as ultraslow optical solitons with TIT [31, 32], enhancement of self-Kerr nonlinearity [33, 34], controlled phase shift up to $\pi/4$ in a single-quantum dot coupled to a photonic crystal nanocavity [35], giant cross-Kerr nonlinearity with spin-orbit coupling [36], and so on [37]. Recently, the realization of giant cross-Kerr nonlinear

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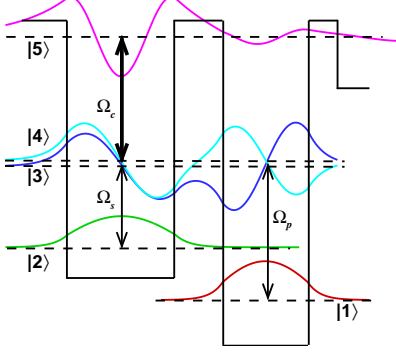


FIG. 1: (color online) Conduction subband of the asymmetric quantum well structure. The solid curves represent the corresponding wave functions.

phase shift and the related quantum information processing (QIP) has been investigated in quantum well (QW) structures based on interband and intersubband transitions [38, 39].

In QW structure, resonant tunneling can induce not only transparency but also large cross-Kerr nonlinearity [40]. However, the nonlinear phase shift on order of π cannot be achieved at single-photon level, since the group velocities of the probe and signal pulses are mismatched [40]. This fact limits its applications in QIP. In the present paper, we suggest an alternative asymmetric QW structure, which combines resonant tunneling and the advantages of inverted-Y-type configuration, and study the linear optical properties and nonlinear optical responses associated with cross-Kerr nonlinearity. It is found, in the present QW structure, that giant cross-Kerr nonlinearity can be achieved with vanishing linear and nonlinear absorptions simultaneously. Consequently, highly entangled photons can be produced and polarization photonic controlled phase gate can be constructed. More importantly, for the probe and signal pulses, the structure is an inherent symmetric configuration. Hence the condition of group velocity matching can be easily satisfied by adjusting the initial electron distribution.

II. STRUCTURE AND LINEAR OPTICAL PROPERTIES

Our asymmetric double QW structure is shown in Fig. 1. The growth sequence of the structure from left to right is as follows. A thick $\text{Al}_{0.50}\text{Ga}_{0.50}\text{As}$ barrier is followed by an $\text{Al}_{0.10}\text{Ga}_{0.90}\text{As}$ layer with thickness of 8.8 nm (shallow well). This shallow well is separated from a 6.9 nm GaAs layer (deep well) on the right by a 3.8 nm $\text{Al}_{0.50}\text{Ga}_{0.50}\text{As}$ potential barrier. Finally, a thin (2.4 nm) $\text{Al}_{0.50}\text{Ga}_{0.50}\text{As}$ barrier separates the deep

well from the last $\text{Al}_{0.40}\text{Ga}_{0.60}\text{As}$ thick layer of on the right. In this structure, one would observe the ground subbands of the right deep well $|1\rangle$ and the left shallow well $|2\rangle$ with energies 57.2 meV and 123.1 meV, respectively. The eigenenergy of the second excited subband of the left shallow well $|5\rangle$ is 385.9 meV. Two new subbands $|3\rangle$ and $|4\rangle$ with eigenenergies 224.1 meV and 231.4 meV are, respectively, created by mixing the first excited subbands of the shallow ($|se\rangle$) and deep ($|de\rangle$) wells by tunneling. Their corresponding wave functions are symmetric and antisymmetric combinations of $|se\rangle$ and $|de\rangle$, i.e., $|3\rangle = (|se\rangle - |de\rangle)/\sqrt{2}$ and $|4\rangle = (|se\rangle + |de\rangle)/\sqrt{2}$. The basic idea is to combine resonant tunneling with the inherited symmetry of invert-Y-type configuration. To do so, we apply a weak probe and a weak signal fields with frequencies ω_p and ω_s to drive the transitions $|1\rangle \leftrightarrow |3\rangle$, $|1\rangle \leftrightarrow |4\rangle$ and $|2\rangle \leftrightarrow |3\rangle$, $|2\rangle \leftrightarrow |4\rangle$, respectively. The subbands $|3\rangle$ and $|4\rangle$ are coupled with $|5\rangle$ by a continuous-wave control field with angular frequency ω_c . Thus, an inverted-Y-type configuration with two-fold degenerate middle subbands is realized. Under the dipole and rotating-wave approximations (RWA), this structure is governed by a set of density matrix equations given below,

$$\dot{\sigma}_{21} = id_{21}\sigma_{21} - i\Omega_p(\sigma_{23} + m\sigma_{24}) + i\Omega_s(\sigma_{31} + q\sigma_{41}), \quad (1)$$

$$\dot{\sigma}_{31} = id_{31}\sigma_{31} + i\Omega_p(\sigma_{11} - \sigma_{33}) - im\Omega_p\sigma_{34} + i\Omega_s\sigma_{21} + i\Omega_c\sigma_{51}, \quad (2)$$

$$\dot{\sigma}_{41} = id_{41}\sigma_{41} + im\Omega_p(\sigma_{11} - \sigma_{44}) - i\Omega_p\sigma_{43} + iq\Omega_s\sigma_{21} + ik\Omega_c\sigma_{51}, \quad (3)$$

$$\dot{\sigma}_{51} = id_{51}\sigma_{51} - i\Omega_p(\sigma_{53} + m\sigma_{54}) + i\Omega_c(\sigma_{31} + k\sigma_{41}), \quad (4)$$

$$\dot{\sigma}_{32} = id_{32}\sigma_{32} + i\Omega_s(\sigma_{22} - \sigma_{33}) + i\Omega_p\sigma_{12} - iq\Omega_s\sigma_{34} + i\Omega_c\sigma_{52}, \quad (5)$$

$$\dot{\sigma}_{42} = id_{42}\sigma_{42} + iq\Omega_s(\sigma_{22} - \sigma_{44}) + im\Omega_p\sigma_{12} - i\Omega_s\sigma_{43} + ik\Omega_c\sigma_{52}, \quad (6)$$

$$\dot{\sigma}_{52} = id_{52}\sigma_{52} - i\Omega_s(\sigma_{53} + q\sigma_{54}) + i\Omega_c(\sigma_{32} + k\sigma_{42}), \quad (7)$$

where $d_{21} = \Delta_p - \Delta_s + i\gamma_{21}$, $d_{31} = \Delta_p + i\gamma_{31}$, $d_{41} = \Delta_p - \delta + i\gamma_{41}$, $d_{51} = \Delta_p + \Delta_c - \Delta_s + i\gamma_{51}$, $d_{32} = \Delta_s + i\gamma_{32}$, $d_{42} = \Delta_s - \delta + i\gamma_{42}$, $d_{52} = \Delta_c + i\gamma_{52}$ with Δ_p , Δ_s , and Δ_c being the detunings of the probe, signal and control fields with the corresponding transitions, and they are defined as $\Delta_{p,(s,c)} = \omega_{p,(s,c)} - (\omega_{3,(3,5)} - \omega_{1,(2,3)})$. $\delta = \omega_4 - \omega_3 \simeq 7.3$ meV denotes the energy difference between the subbands $|3\rangle$ and $|4\rangle$. Halves of the Rabi frequencies of the probe, signal and control fields are $\Omega_p = \vec{\mu}_{13} \cdot \vec{E}_p / 2\hbar$, $\Omega_s = \vec{\mu}_{23} \cdot \vec{E}_s / 2\hbar$, and $\Omega_c = \vec{\mu}_{53} \cdot \vec{E}_c / 2\hbar$ with $\vec{\mu}_{ij}$ being electric dipole momentum between subbands $|i\rangle$ and $|j\rangle$ ($i, j = 1 - 5$ and $i \neq j$), while $m = \mu_{41}/\mu_{31} = -0.73$, $q = \mu_{42}/\mu_{32} = 1.2$, and $k = \mu_{54}/\mu_{53} = 2.3$ give the ratios between the relevant subband transition dipole momentum. E_p , E_s , and E_c are, respectively, the slowly varying electric field amplitudes of the probe, signal and control fields. The half linewidths are, respectively, given by $\gamma_{31} = \gamma_3 + \gamma_{31}^{\text{deph}}$, $\gamma_{41} = \gamma_4 + \gamma_{41}^{\text{deph}}$, $\gamma_{51} = \gamma_5 + \gamma_{51}^{\text{deph}}$, $\gamma_{32} = \gamma_3 + \gamma_{32}^{\text{deph}}$, $\gamma_{42} = \gamma_4 + \gamma_{42}^{\text{deph}}$, $\gamma_{52} = \gamma_5 + \gamma_{52}^{\text{deph}}$. Here

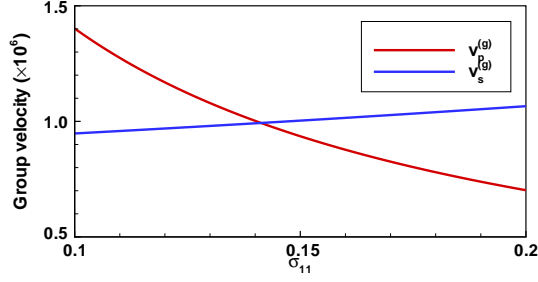


FIG. 2: (color online) The group velocities of the probe and the signal pulses as functions of $\sigma_{11}^{(0)}$ with and without the control field. The parameters are explained in the text.

γ_3 (γ_4 , γ_5) is the electron decay rate of subband $|3\rangle$, ($|4\rangle$, $|5\rangle$) and $\gamma_{ij}^{\text{deph}}$ the electron dephasing rates, which are introduced to account not only for intrasubband phonon scattering and electron-electron scattering but also for inhomogeneous broadening due to scattering on interface roughness. The dipole transition rate from subband $|2\rangle$ to $|1\rangle$ is very small because of the high inter-well barrier between them, $\gamma_{21} \approx \gamma_{21}^{\text{deph}}$. Electron decay rates can be calculated by solving effective mass Schrödinger equation [41]. For temperature up to 10 K and electron

density smaller than 10^{12} cm^{-2} , γ_i^{deph} can be estimated according to Ref. [23].

If all electrons remain in ground subband $|1\rangle$, which means that the signal field drives two virtually empty transitions, the contribution to the susceptibility comes only from higher order. It is hard to achieve group velocity matching since the asymmetry of configuration. Thus, as done in Ref. [36], we assume that electrons distribute not only in subbands $|1\rangle$ but also in $|2\rangle$. The symmetric configuration is hence formed. In the presence of the control field, the subbands $|3, 4\rangle$ and $|5\rangle$ are mixed into three new subbands. The symmetry of scheme ensures that the probe and signal propagate with comparable group velocity. To investigate the group velocities of the probe and signal pulses, following the standard processes [20], we assume $|\Omega_p|, |\Omega_s| \ll |\Omega_c|, \Delta_1, \Delta_2, \Delta_3, \delta$ and solve the density matrix equations (1)-(7) in the nondepletion approximation ($\sigma_{11} + \sigma_{22} \approx 1$) together with Maxwell's equations and expand the linear dispersion relations as Taylor series around their center frequency. The group velocities of the probe and signal pulses are, respectively, given by

$$v_p^g = 1/\text{Re}[K_p^{(1)}], \quad v_s^g = 1/\text{Re}[K_s^{(1)}], \quad (8)$$

with

$$K_p^{(1)} = \frac{1}{c} + \frac{N\omega_p|\mu_{31}|^2\sigma_{11}^{(0)}}{\hbar\epsilon_0 c} \left\{ \frac{[d_{31}d_{41} + d_{31}d_{51} + d_{41}d_{51} - (1+k^2)\Omega_c^2][d_{51}(d_{41} + m^2d_{31}) - (k-m)^2\Omega_c^2]}{[d_{31}d_{41}d_{51} - (d_{41} + k^2d_{31})\Omega_c^2]^2} - \frac{d_{41} + m^2d_{31} + (1+m^2)d_{51}}{d_{31}d_{41}d_{51} - (d_{41} + k^2d_{31})\Omega_c^2} \right\}, \quad (9)$$

$$K_s^{(1)} = \frac{1}{c} + \frac{N\omega_s|\mu_{32}|^2\sigma_{22}^{(0)}}{\hbar\epsilon_0 c} \left\{ \frac{[d_{32}d_{42} + d_{32}d_{52} + d_{42}d_{52} - (1+k^2)\Omega_c^2][d_{52}(d_{42} + m^2d_{32}) - (k-q)^2\Omega_c^2]}{[d_{32}d_{42}d_{52} - (d_{42} + k^2d_{32})\Omega_c^2]^2} - \frac{d_{42} + m^2d_{32} + (1+m^2)d_{52}}{d_{32}d_{42}d_{52} - (d_{42} + k^2d_{32})\Omega_c^2} \right\}, \quad (10)$$

where N is the electron volume density, $\sigma_{11}^{(0)}$ and $\sigma_{22}^{(0)}$ are initial electron distribution in subbands $|1\rangle$ and $|2\rangle$ with $\sigma_{11}^{(0)} + \sigma_{22}^{(0)} = 1$. The dependence of v_p^g and v_s^g on $\sigma_{11}^{(0)}$ shows that it is, in principle, possible to control the group velocities by adjusting the initial electron distribution. The electron decay rates are $\gamma_3 \approx \gamma_4 = 0.5 \text{ meV}$, $\gamma_5 = 0.2 \text{ meV}$ (corresponding intrasubband relaxation time $T_1 \sim 10 \text{ ps}$) [42] and $\gamma_3^{\text{deph}} = \gamma_4^{\text{deph}} = \gamma_5^{\text{deph}} = 0.2 \text{ meV}$ [23]. We take the Rabi frequency and the detuning of the control field as $\Omega_c = 1.5 \text{ meV}$ and $\Delta_c = -5.3 \text{ meV}$. With $\Delta_p = \Delta_s = 3.0 \text{ meV}$ (around the center of their transparency windows) and $N = 5 \times 10^{-17} \text{ cm}^{-3}$, Fig. 2 illustrates the dependence of the group velocities of the probe and signal pulses on the initial electron dis-

tribution $\sigma_{11}^{(0)}$. By controlling the initial electron distribution ($\sigma_{11}^{(0)} \approx 0.141$), the probe and signal pulses will propagate with comparable and small group velocities ($v_p^g = v_s^g \approx 1.0 \times 10^6 \text{ m/s}$). The initial electron distribution can be realized with stimulated Raman adiabatic passage [43].

What we pursue is to produce the strongly interacting and highly entangled photons by virtue of the giant cross-Kerr nonlinearity of the QW structure considered, we hence define the susceptibility as [8]

$$\chi_p = \frac{N|\mu_{13}|^2}{\hbar\epsilon_0} \frac{\sigma_{31} + m\sigma_{41}}{\Omega_p} \approx \chi_p^{(1)} + \chi_p^{(3,\text{SPM})}|E_p|^2 + \chi_p^{(3,\text{XPM})}|E_s|^2, \quad (11)$$

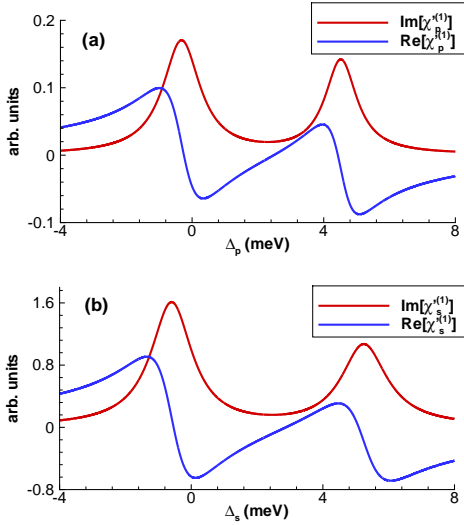


FIG. 3: (color online) The linear absorption (red curve) and dispersion (blue curve) of the probe (a) and the signal fields (b) as functions of their corresponding detunings Δ_p and Δ_s with $\sigma_{11}^{(0)} \approx 0.27$. The detunings are chosen as (a) $\Delta_s = \delta/2 = 3.65$ meV, (b) $\Delta_p = 3.6505$ meV. The other parameters are the same with those in Fig. 2.

$$\chi_s = \frac{N|\mu_{23}|^2}{\hbar\epsilon_0} \frac{\sigma_{32} + q\sigma_{42}}{\Omega_s} \simeq \chi_s^{(1)} + \chi_s^{(3,SPM)}|E_s|^2 + \chi_s^{(3,XPM)}|E_p|^2, \quad (12)$$

where $\chi_{p,s}^{(1)}$, $\chi_{p,s}^{(3,SPM)}$ and $\chi_{p,s}^{(3,XPM)}$ are the linear, self-Kerr, and cross-Kerr susceptibilities of the probe and signal pulses, respectively. By solving the set of density matrix equations (1)-(7) in steady state in the nondepletion approximation, the first and third order susceptibilities associated with cross-Kerr nonlinearity can be calculated ($\chi_{p,s}^{(3,XPM)}$ will be considered in the next section). The linear susceptibilities can be written as

$$\chi_p^{(1)} = \frac{N|\mu_{31}|^2}{\hbar\epsilon_0} \chi_p^{'(1)}, \quad \chi_s^{(1)} = \frac{N|\mu_{32}|^2}{\hbar\epsilon_0} \chi_s^{'(1)}, \quad (13)$$

in which $\chi_p^{'(1)}$ and $\chi_s^{'(1)}$ are given by

$$\chi_p^{'(1)} = -\sigma_{11}^{(0)} \frac{d_{51}(d_{41} + m^2 d_{31}) - (k - m)^2 \Omega_c^2}{d_{31} d_{41} d_{51} - (d_{41} + k^2 d_{31}) \Omega_c^2}, \quad (14)$$

$$\chi_s^{'(1)} = -\sigma_{22}^{(0)} \frac{d_{52}(d_{42} + q^2 d_{32}) - (k - q)^2 \Omega_c^2}{d_{32} d_{42} d_{52} - (d_{42} + k^2 d_{32}) \Omega_c^2}. \quad (15)$$

Equations (14)-(15) show the symmetry of the QW structure between the probe and signal fields. With the simultaneous exchange of $1 \leftrightarrow 2$ and $m \leftrightarrow q$, the expression of $\chi_s^{(1)}$ can be obtained.

The real and imaginary parts of $\chi_p^{'(1)}$ ($\chi_s^{'(1)}$), respectively, account for the linear absorption and dispersion of the probe (signal) field. With $\sigma_{11}^{(0)} \approx 0.141$, their evolutions versus their corresponding detunings are shown in

Figs. 3(a) and (b), respectively. We take $\Delta_s = 3.0$ meV in Fig. 3(a) and $\Delta_p = 2.995$ meV in Fig. 3(b). The other parameters are the same with those in Fig. 2. With this set of parameters, the dispersion of the probe and the signal pulses around the center of the transparent window are linearly proportional to their detunings. This means that the probe and signal pulses will propagate with comparable and small group velocities, and the influence of group velocity dispersion can be neglected within the region considered. At the center of the transparency window, the linear absorptions of the probe and signal fields are very small because of the destructive interference between transition paths, and can be safely ignored.

III. STRONGLY INTERACTING AND HIGHLY ENTANGLED PHOTONS WITH RESONANT TUNNELING

The explicit forms of the probe and signal third order susceptibilities associated with cross-Kerr nonlinearity are given by

$$\chi_p^{(3,XPM)} = \frac{N|\mu_{13}|^2|\mu_{23}|^2}{4\hbar^3\epsilon_0} \chi_p^{'(3)}, \quad (16)$$

$$\chi_s^{(3,XPM)} = \frac{N|\mu_{13}|^2|\mu_{23}|^2}{4\hbar^3\epsilon_0} \chi_s^{'(3)}, \quad (17)$$

where $\chi_p^{'(3)}$ and $\chi_s^{'(3)}$ can be simplified as

$$\chi_p^{'(3)} = -\frac{T_{p1}}{\mathcal{Z}} [T_{p2} + (T_{p3} - T_{p4})\Omega_c^2 + T_{p5}\Omega_c^4], \quad (18)$$

$$\chi_s^{'(3)} = -\frac{T_{s1}}{\mathcal{Z}^*} [T_{s2} + (T_{s3} - T_{s4})\Omega_c^2 + T_{s5}\Omega_c^4], \quad (19)$$

with

$$T_{p1} = d_{51}(d_{41} + mq d_{31}) + (m - k)(k - q)\Omega_c^2,$$

$$T_{p2} = d_{51} d_{25} [\sigma_{11} d_{23} d_{24} (d_{41} + mq d_{31}) - \sigma_{22} d_{31} d_{41} (d_{24} + mq d_{23})],$$

$$T_{p3} = d_{25} [\sigma_{22} (d_{41} + k^2 d_{31}) (d_{24} + mq d_{23}) + (m - k)(k - q) \sigma_{11} d_{23} d_{24}],$$

$$T_{p4} = d_{51} [\sigma_{11} (d_{24} + k^2 d_{23}) (d_{41} + mq d_{31}) + (m - k)(k - q) \sigma_{22} d_{31} d_{41}],$$

$$T_{p5} = (k - m)(k - q) [\sigma_{11} (d_{24} + k^2 d_{23}) - \sigma_{22} (d_{41} + k^2 d_{31})],$$

$$T_{s1} = d_{52} (d_{42} + mq d_{32}) + (m - k)(k - q) \Omega_c^2,$$

$$T_{s2} = d_{52} d_{15} [\sigma_{11} d_{32} d_{42} (d_{14} + mq d_{13}) - \sigma_{22} d_{13} d_{14} (d_{42} + mq d_{32})],$$

$$T_{s3} = d_{15} [\sigma_{11} (d_{42} + k^2 d_{32}) (d_{14} + mq d_{13}) + (m - k)(k - q) \sigma_{22} d_{13} d_{14}],$$

$$T_{s4} = d_{52} [\sigma_{22} (d_{14} + k^2 d_{13}) (d_{42} + mq d_{32}) + (m - k)(k - q) \sigma_{11} d_{32} d_{42}],$$

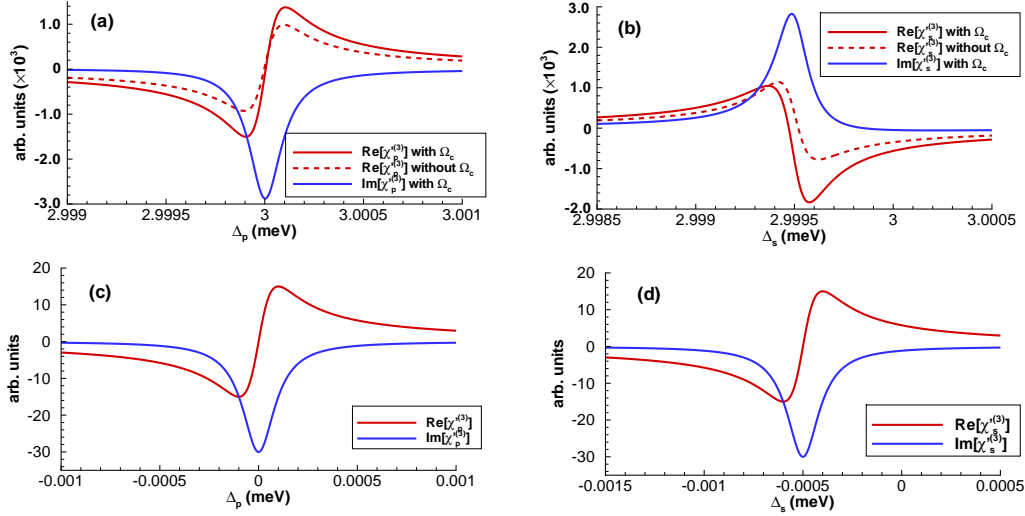


FIG. 4: (color online) (a) $\text{Re}[\chi_p^{(3)}]$ and $\text{Im}[\chi_p^{(3)}]$ versus Δ_p with (solid curves) and without (dashed curves) the control field; (b) $\text{Re}[\chi_s^{(3)}]$ and $\text{Im}[\chi_s^{(3)}]$ versus Δ_s with (solid curves) and without (dashed curves) the control field; (c) $\text{Re}[\chi_p^{(3)}]$ and $\text{Im}[\chi_p^{(3)}]$ versus Δ_p without tunneling by setting $m = q = k = 0$; (d) $\text{Re}[\chi_s^{(3)}]$ and $\text{Im}[\chi_s^{(3)}]$ versus Δ_s without tunneling. The parameters are the same with those in Fig. 3.

$$\begin{aligned}
 T_{s5} &= (k - m)(k - q)[\sigma_{11}(d_{42} + k^2 d_{32}) \\
 &\quad - \sigma_{22}(d_{14} + k^2 d_{13})], \\
 \mathcal{Z} &= d_{21}[d_{23}d_{24}d_{25} - (d_{24} + k^2 d_{23})\Omega_c^2] \\
 &\quad \times [d_{31}d_{41}d_{51} - (d_{41} + k^2 d_{31})\Omega_c^2]^2.
 \end{aligned}$$

The role of resonant tunneling can be seen from the expressions of $T_{\alpha\beta}$ ($\alpha = p, s$, $\beta = 1, 3, 4, 5$). In the QW structure under consideration, the symmetric and asymmetric wave functions of subbands $|3\rangle$ and $|4\rangle$ lead to $m \neq q \neq k$, which indicates that the resonant tunneling can modify the optical nonlinearity such as cross-Kerr effect. In Figs. 4(a) and 4(b), we illustrate the evolutions of $\text{Re}[\chi_{p,s}^{(3)}]$ (represent the cross-Kerr nonlinearities between the probe and signal fields) and $\text{Im}[\chi_{p,s}^{(3)}]$ (account for the nonlinear absorptions) as functions of their corresponding detunings with (solid curves) and without (dashed curves) the control field. All parameters are the same with those in Figs. 3. Within the transparency window, both the strengths of cross-Kerr nonlinearities and nonlinear absorptions of the probe and signal pulses are enhanced dramatically. Fortunately, the probe and signal nonlinear absorption peaks are very sharp, and the real parts of the two cross-Kerr susceptibilities decay much more slowly than their corresponding nonlinear absorptions. We also notice that in the present QW structure, γ_2 is dominantly determined by the electron dephasing rate, smaller γ_2 can be attained by decreasing the temperature. Hence, the cross-Kerr nonlinearities can be enhanced much more than that in Ref. [40]. More importantly, positions of the nonlinear absorption peaks can be controlled by adjusting the detunings Δ_p and Δ_s , which can be seen from Eqs. (18) and (19). For certain detunings, for example $\Delta_p = 2.9995$ meV and

$\Delta_s = 3.0$ meV, we have $\text{Re}[\chi_p^{(3)}] \simeq -419.02 \text{ meV}^{-3}$, $\text{Re}[\chi_s^{(3)}] \simeq -417.12 \text{ meV}^{-3}$, and the two negative cross-Kerr nonlinearities are of the same order of magnitudes. Therefore, giant cross-Kerr nonlinearities are realized, while the nonlinear absorptions can be neglected ($\text{Im}[\chi_p^{(3)}] \simeq -10.1 \text{ meV}^{-3}$ and $\text{Im}[\chi_s^{(3)}] \simeq 8.7 \text{ meV}^{-3}$). As shown in Fig. 4(a) and 4(b), with this set of parameters, the strengths of cross-Kerr nonlinearities can be enhanced with the presence of the control field.

Figures 4(c) and 4(d) illustrate the evolutions of the real and imaginary parts of $\chi_p^{(3)}$ and $\chi_s^{(3)}$ versus their corresponding detunings with $m = q = k = 0$. In this case, the subband $|4\rangle$ is decoupled, and the system can hence be described as an inverted-Y-type configuration. In order to see the effect of resonant tunneling more clearly, we choose $\Delta_c = 0$ and $\Omega_c \approx 2.65$ meV (similar transparency windows as those with tunneling). The other parameters are the same with those in Fig. 4(a) and 4(b). Within the transparency windows, the enhancement of cross-Kerr nonlinearities can still be achieved with vanishing absorptions. However, under the same conditions, the two strengths of cross-Kerr nonlinearity are both much less than those with resonant tunneling. With $\Delta_p = 0.5 \mu\text{eV}$ and $\Delta_s = 0$ meV, we have $\text{Re}[\chi_p^{(3)}] \simeq -5.77 \text{ meV}^{-3}$, $\text{Re}[\chi_s^{(3)}] \simeq 5.77 \text{ meV}^{-3}$. Furthermore, the two cross-Kerr nonlinearities (one positive and one negative) exhibit destructive effect on the conditional phase shift (details will be shown later).

A significant interaction is a very essential requirement for implementation of controlled phase gate between two optical qubits, where the quantum information is stored in the orthogonal polarization degree of freedom. In

the QW structure, by virtue of resonant tunneling, such strong interaction can be realized by the giant cross-Kerr effect. A two-qubit quantum phase gate operation can be expressed as $|i\rangle_p |j\rangle_s \rightarrow \exp(i\phi_{ij}) |i\rangle_p |j\rangle_s$, where $i, j = H, V$ denote the logic qubit basis. The photon-polarization qubit could be realized easily in the QW structure considered, where one photon acquires a phase shift conditioned on the state of another photon. Using polarizing beam splitters (PBS), we assume the interacting scheme shown in Fig. 1 is implemented when the probe and signal fields have $|H\rangle$ polarization [11]. After passing through a PBS, the vertically polarized component of each photon is transmitted, while the horizontally polarized component is directed into the QW structure, wherein the two-photon state $|H\rangle_p |H\rangle_s$ acquires the conditional phase shift. At the output, each photon is recombined with its vertically polarized component on another PBS. We assume the probe and signal polarized single-photon wave packets can be expressed as

$$|\psi_{p,s}\rangle = (|H\rangle_{p,s} + |V\rangle_{p,s})/\sqrt{2}, \quad (20)$$

which can be written as $|H, V\rangle_{p,s} = \int d\omega \xi_{p,s}(\omega) a_{H,V}^\dagger(\omega) |0\rangle$ with $\xi_{p,s}(\omega)$ being Gaussian frequency distribution of incident wave packets centered on $\omega_{p,s}$. Then the polarization phase gate truth table goes as

$$|V\rangle_p |V\rangle_s = e^{-i(\phi_p^0 + \phi_s^0)} |V\rangle_p |V\rangle_s, \quad (21)$$

$$|H\rangle_p |V\rangle_s = e^{-i(\phi_p^l + \phi_s^0)} |H\rangle_p |V\rangle_s, \quad (22)$$

$$|V\rangle_p |H\rangle_s = e^{-i(\phi_p^0 + \phi_s^l)} |V\rangle_p |H\rangle_s, \quad (23)$$

$$|H\rangle_p |H\rangle_s = e^{-i(\phi_p^l + \phi_s^l + \phi_p^n + \phi_s^n)} |H\rangle_p |H\rangle_s, \quad (24)$$

in which $\phi_{p,s}^0 = k_{p,s}l$ denotes the trivial vacuum phase shift with l being the length of the QW structure, $\phi_{p,s}^l = 2\pi k_{p,s}l \text{Re}[\chi_{p,s}^{(1)}]$ linear phase shift, and $\phi_{p,s}^n$ the probe and signal nonlinear phase shift induced by cross-Kerr effect. The conditional phase shift can be written as $\phi_t^n = \phi_p^n + \phi_s^n$, which is only attributed by the cross-Kerr nonlinearity. Provided the conditional phase shift is not zero, the above set of equations support a universal quantum phase gate [7]. For Gaussian probe and signal pulses of time durations $\tau_{p,s}$, and with peak Rabi frequencies $\Omega_{p,s}^0$, solving the propagation equations gives the nonlinear cross-phase shift $\phi_{p,s}^n$ [8]

$$\phi_p^n = \frac{2\omega_p l}{c} \frac{\hbar^2 |\Omega_s^0|^2}{|\mu_{23}|^2} \frac{\text{erf}(\zeta_p)}{\zeta_p} \text{Re}[\chi_p^{(3)}], \quad (25)$$

$$\phi_s^n = \frac{2\omega_s l}{c} \frac{\hbar^2 |\Omega_p^0|^2}{|\mu_{13}|^2} \frac{\text{erf}(\zeta_s)}{\zeta_s} \text{Re}[\chi_s^{(3)}], \quad (26)$$

where $\zeta_p = [(1 - v_g^p/v_g^s)\sqrt{2}l]/(v_g^p\tau_s)$, and ζ_s can be obtained from ζ_p upon interchanging $p \leftrightarrow s$. $\text{erf}(\zeta)$ represents the error function. The nonlinear phase shift acquired by the probe and signal pulses propagating through the QW structure can be controlled by the signal and probe pulses intensity. In Figs. 5 (a) and (b),

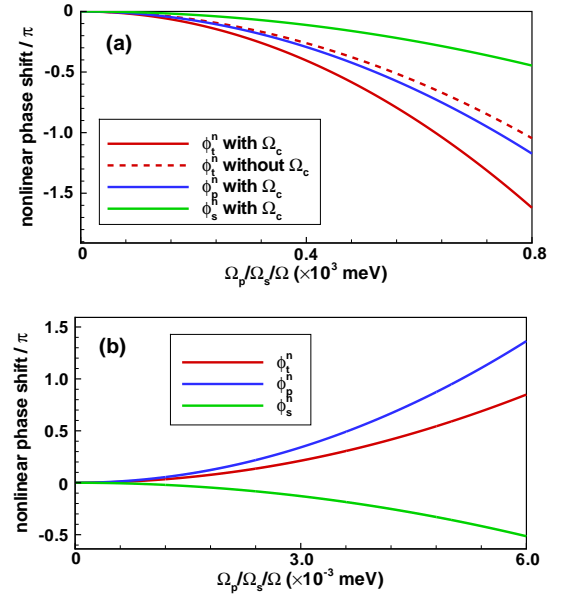


FIG. 5: (color online) (a) ϕ_p^n , ϕ_s^n , and ϕ_t^n with and without the control field versus the Rabi frequency $\Omega_p = \Omega_s = \Omega$; (b) ϕ_p^n , ϕ_s^n , and ϕ_t^n versus the Rabi frequency $\Omega_p = \Omega_s = \Omega$ with $m = q = k = 0$. The length of QWs is taken as 1.0 mm, and the other parameters are same as those in Fig. 3.

we plot the evolutions of the probe nonlinear phase shift ϕ_p^n , the signal nonlinear phase shift ϕ_s^n , and the conditional phase shift ϕ_t^n as functions of $\Omega_p^0 = \Omega_s^0 = \Omega$ with (a) and without (b) resonant tunneling. The length of QW structure is taken as $l = 1.0$ mm. With resonant tunneling, $\text{Re}[\chi_p^{(3,\text{XPM})}] \cdot \text{Re}[\chi_s^{(3,\text{XPM})}] > 0$ leads to $\phi_p^n \cdot \phi_s^n > 0$, which indicates the constructive effect of the probe and signal nonlinear phase shift on the conditional phase shift (see Fig. 5(a)). $\phi_t^n = \pi$ can be achieved with $\Omega_p = \Omega_s = \Omega \approx 6.22 \times 10^{-4}$ meV. The probe and signal pulses can have a mean amplitude of about one photon when these beams are focused or propagate in a tightly confined waveguide. With these parameters, the corresponding intensities of the probe and the signal pulses are, respectively, given by $I_p \approx 4.35$ mW cm $^{-2}$ and $I_s \approx 6.94$ mW cm $^{-2}$. We remark that the intensities of a single probe and signal photons per 0.1 ns on the area of $1 \mu\text{m}^2$ are $I_p \approx 27.3$ mW cm $^{-2}$ and $I_s \approx 22.5$ mW cm $^{-2}$, respectively. The numerical findings indicate that our semiconductor QW structure can indeed make a polarization photonic controlled phase gate with a π -conditional phase shift possible with single-photon wave packets. In Fig. 5(a), we also illustrate the positive effect of the control field on the conditional phase shift. Without resonant tunneling, $\text{Re}[\chi_p^{(3,\text{XPM})}] \cdot \text{Re}[\chi_s^{(3,\text{XPM})}] < 0$ exhibits the destructive effect on conditional phase shift (see Fig. 5(b)). In this case, the conditional phase shift on order of π can be obtained with more than one photons, i.e., $\Omega_p = \Omega_s = \Omega \approx 6.5 \times 10^{-3}$ meV ($I_p \approx 0.48$ W cm $^{-2}$ and $I_s \approx 0.76$ W cm $^{-2}$).

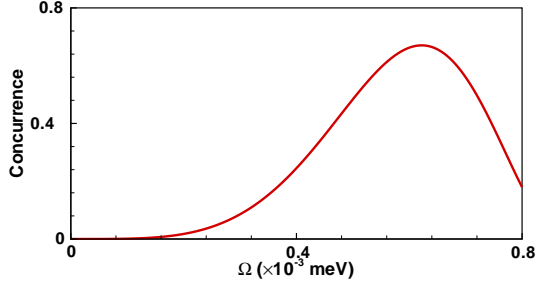


FIG. 6: (color online) The concurrence versus the Rabi frequency $\Omega_p = \Omega_s = \Omega$. The other parameters are same as those in Fig. 3.

We now turn to the problem of entangled-photon states in our QW structure by using the entanglement of formation. Starting from the truth table (Eqs. (21)-(24)), the degree of entanglement of two polarized photons state can be computed. For an arbitrary two-qubit system, the degree of entanglement is defined by [44]

$$E_F(C) = h\left(\frac{1 + \sqrt{1 - C^2}}{2}\right), \quad (27)$$

where $h(x) = -x \log_2(x) - (1-x) \log_2(1-x)$ is Shannon's entropy function, and concurrence C is given by

$$C(\hat{\rho}) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \quad (28)$$

with λ_i ($i = 1-4$) being the square roots of the eigenvalues of $\hat{\rho}(\hat{\sigma}_y^p \otimes \hat{\sigma}_y^s)\hat{\rho}^*(\hat{\sigma}_y^p \otimes \hat{\sigma}_y^s)$ in descending order. Here $\hat{\rho}^*$ denotes the complex conjugation of the output state density matrix $\hat{\rho}$, and $\hat{\sigma}_y^j$ ($j = p, s$) is the y -component of the Pauli matrix. The concurrence C can be taken as a kind of measure of entanglement since $E_F(C)$ is a monotonic increasing function of C . As an example, we plot the evolution of the concurrence versus the Rabi frequency $\Omega_p = \Omega_s = \Omega$ in Fig. 6. With the set of parameters in Fig. 5(a), the maximum degree of entanglement obtained in our proposal can be as large as $E_F \approx 0.55$ (in Ref. [38], $E_F \approx 0.35$) with $\Omega \approx 6.22 \times 10^{-4}$ meV, corresponding to the point of the conditional phase shift $\phi_t^n \approx \pi$.

IV. CONCLUSIONS

In the literature several semiconductor QW structures have been suggested to investigate the possibility of quantum phase gate [38, 39]. We should point out that, in asymmetrical N-type [38] or ladder [39] configurations,

the probe pulse propagates with slow light because of EIT, while the signal pulse possesses a nonzero Kerr nonlinearity only. The group velocity matching can only be satisfied by controlling the signal detuning when the signal field is continuous wave. However, this is not desirable for photonic controlled phase gate. This is a consequence of the asymmetry configurations. In the present study, within the transparency window considered, the influence of group velocity dispersion can be ignored safely. That is to say, it is possible that the probe and signal wave packets propagate in QW structure with group velocity matching and higher stability.

In conclusion, we have designed a double QW structure to achieve strongly interacting and highly entangled photons. This structure combines the resonant tunneling with the advantages of inverted-Y type scheme. By virtue of resonant tunneling, not only the strength of cross-Kerr nonlinearities can be enhanced dramatically with vanishing linear and nonlinear absorptions simultaneously, but also the effect of cross-Kerr nonlinearities of the probe and signal pulses on the conditional phase shift can be changed from destructive to constructive. Our numerical findings confirm that it is possible to achieve nonlinear phase shift on order of π at a single photon level. Based on such important features, we have demonstrated that it is possible to produce highly entangled photon pairs and construct polarization qubit quantum phase gates. For the probe and the signal pulses, the interacting scheme is symmetric, and thus yields equal group velocities by adjusting the initial distribution of electron. We believe that the present study may be useful for guiding experimental realization of electrooptically modulated devices and facilitating more practical applications in solid quantum information processing.

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